You already know a lot about quadratic functions and you have made several connections between the different representations of quadratics.  Today you are going to develop a method to sketch a parabola from its equation without a table.

* **8-64.** WHAT DO YOU NEED TO SKETCH A PARABOLA?

How many points do you need in order to sketch a parabola?  One?  Ten?  Fifty?  Think about this as you answer the questions below.  (Note: A sketch does not need to be exact.  The parabola merely needs to be reasonably placed with important points clearly labeled.)

* 1. Can you sketch a parabola if you only know where its *y*-intercept is?  For example, if the *y*-intercept of a parabola is at (0, –15), can you sketch its graph?  Why or why not?
  2. What about the two *x-*intercepts of the parabola?  If you only know where the *x‑*intercepts are, can you draw the parabola?  For example, if the *x-*intercepts are at (–3, 0) and (5, 0), can you predict the path of the parabola?
  3. Can you sketch a parabola with only its *x*-intercepts and *y*‑intercept?  To test this idea, sketch the graph of a parabola for the quadratic equation y = *x*2 − 2*x* − 15 with roots (−3, 0) and (5, 0) and *y*-intercept (0, −15).
  4. Where is the line of symmetry?
* **8-65.** In problem 8-64, you learned that if you can find the intercepts of a parabola from a rule, then you can sketch the graph without a table.

1. What is true about the value of *y* for all *x-*intercepts?  What is true about the value of *x* for all *y*-intercepts?
2. If *x* = 0 at the *y*-intercept, find the *y*-intercept of *y* = 2*x*2 + 5*x* − 12.
3. Since the *x-*intercept occurs when *y* = 0, write the equation that you would need to solve to find the *x*-intercepts for the graph of  *y* = 2*x*2 + 5*x* − 12.
4. The **roots** or **zeros** of a quadratic expression are the values of x that make the value of the quadratic equal to zero.  An *x*-intercept of a quadratic function is a root.  At this point, can you solve 2*x*2 + 5*x* − 12 = 0 for *x*?

* **8-66.** ZERO PRODUCT PROPERTY

The equation you wrote in part (c) of problem 8‑65 is called a **quadratic equation**.  To solve it, you need to examine what you know about zero.  Study the special properties of zero below.

* Nathan, Sonia, and Gaston are playing a game where Nathan and Sonia each think of a number and then give Gaston a clue about their numbers.  Using the clue, Gaston must tell them everything that he knows about their numbers.

1. Nathan and Sonia’s first clue for Gaston is that when you multiply their numbers together, the result is zero.  What conclusion can Gaston make?
2. Disappointed that Gaston came so close to figuring out their numbers, Nathan and Nancy invite Nadia over to make things harder. Nathan, Nancy, and Nadia all think of secret numbers. This time Gaston is told that when their *three* secret numbers are multiplied together, the answer is zero. What can Gaston conclude this time?
3. Does it matter how many numbers are multiplied?  If the product is zero, what do you know about one of the numbers?  This property is called the **Zero Product Property**.  Write a description of this property below.

* **8-67.** Let’s investigate how can you use the Zero Product Property to help you solve the quadratic equation 2*x*2 + 5*x* − 12 from part (d) of problem 8-65?

1. Examine the quadratic equation.  Is there a product that equals zero?  If not, how can you rewrite the quadratic expression as a product?

1. Now that the equation is written as a product of factors equaling zero, you can use the Zero Product Property to solve it.  Since you know that one of the factors must be zero, you can set up two simpler equations to help you solve for *x*.  Use one factor at a time and determine what *x*-value makes it equal to zero.
2. What do these solutions represent?  What do they tell you?
3. You now know the roots (also called the zeros) of  2*x*2 + 5*x* − 12. Use the roots to find the x-intercepts of the graph of the parabola 2*x*2 + 5*x* − 12. Next, find the y-intercept. Then sketch a graph of the parabola.

* **8-68.** Can you make a sketch of a parabola from the two *x*-intercepts and the *vertex*?

1. Find the *x*-intercepts of the parabola *y* = *x*2 − 2*x* − 8.
2. Make a prediction of where the vertex would be and explain why.

1. Sketch the graph.

1. Which was easier, using the vertex or using the *y*-intercept to draw the parabola?