



Practice

13.2 Angles of Rotation

For each angle below, find all coterminal angles, θ , such that $-360^\circ < \theta < 360^\circ$. Then find the corresponding reference angle.

- | | | |
|-----------------------|-----------------------|----------------------|
| 1. 47° _____ | 2. -123° _____ | 3. 218° _____ |
| 4. 512° _____ | 5. -222° _____ | 6. 307° _____ |
| 7. 1122° _____ | 8. -185° _____ | 9. 645° _____ |

Find the reference angle.

- | | | |
|------------------------|------------------------|------------------------|
| 10. 105° _____ | 11. -213° _____ | 12. 715° _____ |
| 13. -144° _____ | 14. 860° _____ | 15. -72° _____ |
| 16. -2° _____ | 17. 1000° _____ | 18. -420° _____ |

Find the exact values of the six trigonometric functions of θ , given each point on the terminal side of θ in standard position.

- | | | |
|-----------------------|-----------------------|-----------------------|
| 19. (12, 8) | 20. (-5, 10) | 21. (4, 9) |
| $\sin \theta =$ _____ | $\sin \theta =$ _____ | $\sin \theta =$ _____ |
| $\cos \theta =$ _____ | $\cos \theta =$ _____ | $\cos \theta =$ _____ |
| $\tan \theta =$ _____ | $\tan \theta =$ _____ | $\tan \theta =$ _____ |
| $\csc \theta =$ _____ | $\csc \theta =$ _____ | $\csc \theta =$ _____ |
| $\sec \theta =$ _____ | $\sec \theta =$ _____ | $\sec \theta =$ _____ |
| $\cot \theta =$ _____ | $\cot \theta =$ _____ | $\cot \theta =$ _____ |

Given the quadrant of θ in standard position and a trigonometric function value of θ , find exact values for the indicated trigonometric function.

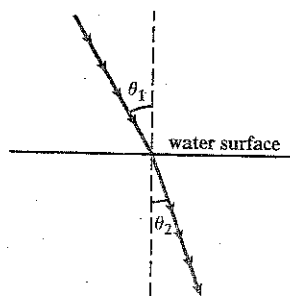
- | | | |
|--|--|--|
| 22. IV, $\sin \theta = -\frac{3}{5}$; $\tan \theta$ _____ | 23. I, $\tan \theta = \frac{5}{8}$; $\csc \theta$ _____ | 24. II, $\cos \theta = -\frac{5}{8}$; $\sin \theta$ _____ |
| 25. III, $\csc \theta = -1.25$; $\tan \theta$ _____ | 26. II, $\cot \theta = -2.4$; $\sin \theta$ _____ | 27. IV, $\sec \theta = \frac{4}{3}$; $\cot \theta$ _____ |

76. $\cos \theta = -0.3420$ and the terminal side of θ lies in quadrant III.
 77. $\tan \theta = -0.8391$ and the terminal side of θ lies in quadrant II.
 78. $\tan \theta = 11.4301$ and the terminal side of θ lies in quadrant III.
 79. $\sin \theta = -0.3420$ and the terminal side of θ lies in quadrant IV.
 80. $\sin \theta = -0.4226$ and the terminal side of θ lies in quadrant III.

APPLICATIONS

In Exercises 81–84, refer to the following:

When light passes from one substance to another, such as from air to water, its path bends. This is called *refraction* and is what is seen in eyeglass lenses, camera lenses, and gems. The rule governing the change in the path is called *Snell's Law*, named after a Dutch astronomer: $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where the n_1 and n_2 are the indices of refraction of the different substances and the θ_1 and θ_2 are the respective angles that light makes with a line perpendicular to the surface at the boundary between substances. The figure shows the path of light rays going from air to water. Assume that the index of refraction in air is 1.



81. **Optics.** If light rays hit the water's surface at an angle 30° from the perpendicular and are refracted to an angle of 22° from the perpendicular, then what is the refraction index for water?
 82. **Optics.** If light rays hit a glass surface at an angle 30° from the perpendicular and are refracted to an angle of 18° from the perpendicular, then what is the refraction index for glass?

CATCH THE MISTAKE

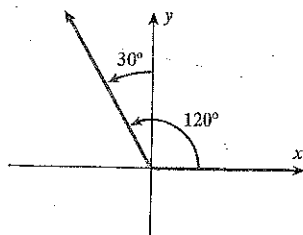
In Exercises 89 and 90, explain the mistake that is made.

89. Evaluate the expression $\sec 120^\circ$ exactly.

Solution:

120° lies in quadrant II.

The reference angle is 30° .



83. **Optics.** If the refraction index for a diamond is 2.4, then to what angle is light refracted if it enters the diamond at an angle of 30° ?

84. **Optics.** If the refraction index for a rhinestone is 1.9, then to what angle is light refracted if it enters the rhinestone at an angle of 30° ?

85. **Merry-Go-Round.** Penelope loves to ride merry-go-rounds. Ben models her path on a Cartesian plane, with the pole of the merry-go-round at the origin and θ being the angle between the positive x-axis and the ray from the origin through her current position. When Penelope gets on, her position makes $\theta = 30^\circ$. If Penelope continues counterclockwise around the merry-go-round for $3\frac{1}{4}$ revolutions, what is the value for the sine of the new value of θ ?

86. **Merry-Go-Round.** Given the model of Penelope on the merry-go-round in Exercise 85, consider the following. When Penelope gets on the merry-go-round, $\theta = 120^\circ$. She rides clockwise for $\frac{2}{3}$ of a revolution and then stops and rides in the opposite direction for $\frac{3}{4}$ revolution before getting off. What is the cosine of the angle given by her new position?

87. **Clock.** Let α be the angle formed by a ray from the center of a clock through the 3 and the clock's minute hand. If $\tan \alpha = 0$, at what number is the minute hand pointing?

88. **Clock.** Consider α as given in Exercise 87. If $\sin \alpha = \frac{1}{2}$, at what number is the minute hand pointing?

Find the cosine of the reference angle.

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

Cosine is negative in quadrant II.

$$\cos 120^\circ = -\frac{\sqrt{3}}{2}$$

Secant is the reciprocal of cosine.

$$\sec 120^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

This is incorrect. What mistake was made?

90. Find the sine to the nearest side of θ (in

Solution:

Evaluate with calculator.

Approximate nearest degree

This is incor

CONCEPT

In Exercises 91–95, or false.

91. It is possible same angle
 92. It is possible same angle
 93. The trigonometric negative
 94. The trigonometric positive mea

CHALLENGE

97. Name the reference angle
 $\cos \theta = -\frac{\sqrt{3}}{2}$
 98. Find $\tan x$ if

TECHNOLOGY

101. Use a calculator to calculate the positive, in calculator a
 102. Use a calculator to calculate the negative, calculator a
 103. Use a calculator to calculate the positive, calculator a