EXAMPLE 3 Writing Trigonometric Function Values in Terms of Their Cofunctions

Write each function or function value in terms of its cofunction.

b.
$$tan x$$

Solution (a):

$$\sin\theta = \cos(90^\circ - \theta)$$

Substitute
$$\theta = 30^{\circ}$$
.

$$\sin 30^{\circ} = \cos (90^{\circ} - 30^{\circ})$$

$$\sin 30^\circ = \cos 60^\circ$$

Solution (b):

Cotangent is the cofunction of tangent.

$$\tan\theta = \cot(90^{\circ} - \theta)$$

Substitute
$$\theta = x$$
.

$$\tan x = \cot(90^\circ - x)$$

Solution (c):

Secant is the cofunction of cosecant.

$$\csc\theta = \sec(90^{\circ} - \theta)$$

Substitute
$$\theta = 40^{\circ}$$
.

$$\csc 40^{\circ} = \sec (90^{\circ} - 40^{\circ})$$

$$csc40^{\circ} = sec50^{\circ}$$

Simplify.

$\csc 40^{\circ} = \sec 50^{\circ}$

a. $\sin 45^{\circ}$ **b.** $\sec (90^{\circ} - y)$

YOUR TURN

Write each function or function value in terms of its cofunction.

a. cos 45°

b. cscy

SECTION

Answer:

In this section we have defined trigonometric functions as ratios of the lengths of sides of right triangles. This approach is called right triangle trigonometry. This is the first of three definitions of trigonometric functions (others will follow in Chapters 2 and 3). We now can find trigonometric functions of an acute angle by taking ratios of the three sides of a right triangle: "adjacent," "opposite," and "hypotenuse."

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
 $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

It is important to remember that "adjacent" and "opposite" are with respect to one of the acute angle we are considering. We learned that trigonometric functions of an angle are equal to the cofunctions of the complement to the angle.



In Exercises 1-6, refer to the triangle in the drawing to find the indicated trigonometric function values.

1. $\sin\theta$

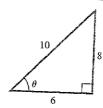
2. $\cos\theta$

3. csc θ

4. $\sec \theta$

5. $tan \theta$

6. $\cot \theta$



In Exercises denominator

7. $\cos\theta$

10. $\csc \theta$

In Exercises

13. sin 60°

16. $\cot A =$

In Exercise:

19. $\sin(x +$

22. cos(A -1

For Exercis

A man lives to the groce whether to vehicle (AT ATV, but th distance he

> 25. Shorte car along t would he I answer to 1

26. Shorte streets, it v have to go nearest yar

27. Roofu garage is f vertically, angle at th and a hori

In Exercises 7-12, refer to the triangle in the drawing to find the indicated trigonometric function values. Rationalize any denominators containing radicals that you encounter in the answers.

7. $\cos\theta$

8. $\sin\theta$

9. $\sec \theta$

10. $\csc\theta$

11. $\tan \theta$

12. $\cot \theta$



In Exercises 13-18, use the cofunction identities to fill in the blanks.

- 13. $\sin 60^{\circ} = \cos$
- 14. $\sin 45^{\circ} = \cos$
- 15. $\cos x = \sin$

- 16. $\cot A = \tan$
- 17. $\csc 30^{\circ} = \sec$ _____
- **18.** $\sec B = \csc$ _____

In Exercises 19-24, write the trigonometric function values in terms of its cofunction.

19. $\sin(x + y)$

20. $\sin(60^{\circ} - x)$

21. $\cos(20^{\circ} + A)$

22. $\cos(A + B)$

function.

"opposite" are

onsidering. We

are equal to the

23. $\cot(45^{\circ} - x)$

24. $\sec(30^{\circ} - \theta)$

DATE NO ATRIONS

For Exercises 25 and 26, consider the following scenario:

A man lives in a house that borders a pasture. He decides to go to the grocery store to get some milk. He is trying to decide whether to drive along the roads in his car or take his all terrain vehicle (ATV) across the pasture. His car drives faster than the ATV, but the distance the ATV would travel is less than the distance he would travel in his car.



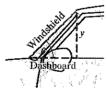
- 25. Shortcut. If $\sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$ and if he drove his car along the streets, it would be 14 miles round trip. How far
- **26.** Shortcut. If $\tan \theta = 1$ and if he drove his car along the streets, it would be 200 yards round trip. How far would he have to go on his ATV round trip? Round your answer to the nearest yard.

would he have to go on his ATV round trip? Round your

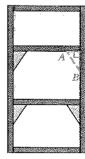
answer to the nearest mile.

27. Roofing. Bob is told that the pitch on the roof of his garage is 5-12, meaning that for every 5 feet the roof increases vertically, it increases 12 feet horizontally. If θ is defined as the angle at the corner of the roof formed by the pitch of the roof and a horizontal line, what is $\sin \theta$?

- **28. Roofing.** Bob's roof has a 5-12 pitch while his neighbor's roof has a 7-12 pitch. With θ defined as the angle formed at the corner of the roof by the pitch of the roof and a horizontal line, whose roof has a larger value for $\cos \theta$? Explain.
- **29.** Windshield. The angle, θ , formed by a car's windshield and dashboard is such that $\tan \theta = \sqrt{3}$. What is the measure of angle θ ?



- **30.** Windshield. Consider the information related to the interior of the car given in Exercise 29. If the vertical distance y between the top of the windshield and the horizontal plane of the dashboard is 3 ft, what is the length of the windshield?
- Bookshelves. Juan is building a bookcase. To give each shelf extra strength, he is planning to add a triangle shaped brace on each end. If each brace is a right triangle as shown below, such that $\cos A = \frac{2}{5}$, find $\sin B$.



32. Bookshelves. If the height of the brace shown in Exercise 31 is 8 inches, find the width of the brace.