

78 CHAPTER 2 Trigonometric Functions

In Exercises 33–38, match each of the angles (33–38) with its coterminal angle (a–f).

- a. 30° b. -95° c. 185° d. -560° e. 780° f. 75°

33. -535° 34. -690° 35. 60° 36. 265° 37. -645° 38. 160°

In Exercises 39–50, determine the angle of the smallest possible positive measure that is coterminal with each of the following angles.

39. 412° 40. 379° 41. -92° 42. -187° 43. -390° 44. 945°
 45. 510° 46. 1395° 47. 154° 48. 360° 49. -1050° 50. 2631°

APPLICATIONS

51. Clock. What is the measure of the angle swept out by the second hand if it starts on the 3 and continues for 3 minutes and 20 seconds?

52. Clock. What is the measure of the angle swept out by the hour hand if it starts at 3 P.M. on Wednesday and continues until 5 P.M. on Thursday.

53. Tetherball. Joe and Alexandria are playing a game of tetherball. Alexandria begins the game and serves the ball counterclockwise. After traveling $3\frac{1}{2}$ revolutions, the ball is struck by Joe in a clockwise direction. If the path of the ball is modeled on a Cartesian plane with the initial position of the ball at 0° , at what angle is the ball 2 revolutions after Joe hits it?

54. Tetherball. If the game of tetherball described in Exercise 53 is won when one player hits the ball in his or her direction through 6 revolutions, through what angle must the ball be hit to win the game?

55. Track. Don and Ron both started running around a circular track, starting at the same point. But Don ran counterclockwise and Ron ran clockwise. The paths they ran swept through angles of 900° and -900° , respectively. Did they end up in the same spot when they finished?

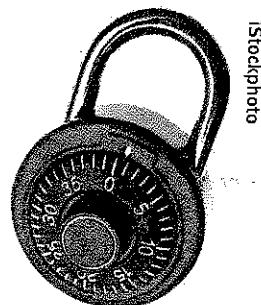
56. Track. Dan and Stan both started running around a circular track, starting at the same point. The paths they ran swept through angles of 3640° and 1890° , respectively. Did they end up in the same spot when they finished?

For Exercises 57 and 58, refer to the following:

A common school locker combination lock is shown. The lock has a dial with 40 calibration marks numbered 0 to 39. A combination consists of 3 of these numbers (e.g., 5-35-20). To open the lock, the following steps are taken:

- Turn the dial clockwise 2 full turns.
- Continue turning clockwise until the first number of the combination is reached.
- Turn the dial counterclockwise one full turn.
- Continue turning counterclockwise until the 2nd number is reached.

- Turn the dial clockwise again until the 3rd number is reached.
- Pull the shank and the lock will open.



Stockphoto

57. Combination Lock. Given that the initial position of the dial is at zero (shown in the picture), how many degrees is the dial rotated in total (sum of clockwise and counterclockwise rotations) to open the lock if the combination is 35-5-20?

58. Combination Lock. Given that the initial position of the dial is at zero (shown in the picture), how many degrees is the dial rotated in total (sum of clockwise and counterclockwise rotations) to open the lock if the combination is 20-15-5?

59. Kite. Henri is flying a kite on the beach. He lets out 100 feet of string and has it flying at an angle of 60° to the ground. How far is the kite extended horizontally and vertically from Henri?

60. Kite. Camille flies her kite at an angle of 45° to the ground. If she has used 75 feet of string, how far is the kite extended horizontally and vertically from Camille?

61. Dartboard. If a dartboard is superimposed on a Cartesian plane, in what quadrant or on what axis does a dart land if its position is given by the point $(-3, 5)$?

62. Dartboard. If a dartboard is superimposed on a Cartesian plane, in what quadrant or on what axis does a dart land if its position is given by the point $(0, -2)$?

63. Ferris Wheel. 1200 feet in diameter, Cartesian plane. If it rotates through 5 full

CATCH UP

In Exercises 65 and

65. Find the angle coterminal with that both angles

Solution:

Coterminal angle complementary

Add 45° to both. This is incorrect

CONCEPT

In Exercises 67–70 true or false.

67. The terminal side of an angle is in the same quadrant as the angle.

68. An acute angle is a standard position angle.

69. If the measure of an angle is 0° , the angle is coterminal with 360° .

70. The difference between two angles must be a multiple of 360° .

CHALLENGE

73. Write an expression for the angle

74. How many angles are there that -2000° is coterminal with?

The following table summarizes the trigonometric function values for the quadrantal angles 0° , 90° , 180° , 270° , and 360° .

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\cot \theta$	$\sec \theta$	$\csc \theta$
0°	0	1	0	undefined	1	undefined
90°	1	0	undefined	0	undefined	1
180°	0	-1	0	undefined	-1	undefined
270°	-1	0	undefined	0	undefined	-1
360°	0	1	0	undefined	1	undefined

SECTION 2.2 SUMMARY

Trigonometric functions are defined in the Cartesian plane as ratios of coordinates and distances.

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad r = \sqrt{x^2 + y^2}$$

Right triangle trigonometric definitions learned in Chapter 1 are consistent with these definitions. We now have the ability to evaluate trigonometric functions for nonacute angles. Trigonometric functions are not always defined for quadrantal angles.

SECTION 2.2 EXERCISES

SKILLS

In Exercises 1–16, the terminal side of an angle θ in standard position passes through the indicated point. Calculate the values of the six trigonometric functions for angle θ .

- | | | | |
|---------------------------------|---------------------------------|------------------------------------|------------------------------------|
| 1. (1, 2) | 2. (2, 3) | 3. (3, 6) | 4. (8, 4) |
| 5. $(\frac{1}{2}, \frac{2}{3})$ | 6. $(\frac{4}{7}, \frac{2}{3})$ | 7. (-2, 4) | 8. (-1, 3) |
| 9. (-4, -7) | 10. (-9, -5) | 11. $(-\sqrt{2}, \sqrt{3})$ | 12. $(-\sqrt{3}, \sqrt{2})$ |
| 13. $(-\sqrt{5}, -\sqrt{3})$ | 14. $(-\sqrt{6}, -\sqrt{5})$ | 15. $(-\frac{10}{3}, \frac{4}{3})$ | 16. $(-\frac{2}{9}, -\frac{1}{3})$ |

In Exercises 17–24, calculate the values for the six trigonometric functions of the angle θ given in standard position, if the terminal side of θ lies on the given line.

- | | | | |
|--|--|---------------------------------------|---------------------------------------|
| 17. $y = 2x \quad x \geq 0$ | 18. $y = 3x \quad x \geq 0$ | 19. $y = \frac{1}{2}x \quad x \geq 0$ | 20. $y = \frac{1}{2}x \quad x \leq 0$ |
| 21. $y = -\frac{1}{3}x \quad x \geq 0$ | 22. $y = -\frac{1}{3}x \quad x \leq 0$ | 23. $2x + 3y = 0 \quad x \leq 0$ | 24. $2x + 3y = 0 \quad x \geq 0$ |

In Exercises 25–40, calculate (if possible) the values for the six trigonometric functions of the angle θ given in standard position.

- | | | | |
|---------------------------|---------------------------|---------------------------|---------------------------|
| 25. $\theta = 450^\circ$ | 26. $\theta = 540^\circ$ | 27. $\theta = 630^\circ$ | 28. $\theta = 720^\circ$ |
| 29. $\theta = -270^\circ$ | 30. $\theta = -180^\circ$ | 31. $\theta = -90^\circ$ | 32. $\theta = -360^\circ$ |
| 33. $\theta = -450^\circ$ | 34. $\theta = -540^\circ$ | 35. $\theta = -630^\circ$ | 36. $\theta = -720^\circ$ |
| 37. $\theta = 810^\circ$ | 38. $\theta = 900^\circ$ | 39. $\theta = -810^\circ$ | 40. $\theta = -900^\circ$ |

APPLICA

In Exercises 41 and

41. **Geometry.** A line passes through the x-axis and its terminal side is in standard position. Find $\tan \theta$.

42. **Geometry.** A line passes through the x-axis and its terminal side is in standard position. Find $\cos \theta$.

In Exercises 43 and

CATCH

In Exercises 49 and

49. The terminal side of an angle θ passes through the point $(-3, 4)$.

Solution:

Label the coordinates x and y . Calculate r .

Use the definition of the trigonometric functions.

Substitute x , y , and r into the formulas.

This is incorrect.